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SCHEDULE PLANNING FOR TRANSIT OPERATIONS

by

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ABSTRACT

The purpose of this paper is to briefly review and comment on the state of the art of schedule planning for transit and paratransit operations. The methods used in practice are broadly reviewed and references for detailed descriptions are given. We examine in turn the problems of time-tabling, vehicle scheduling and crew scheduling for transit operation. Computer scheduling for paratransit is described in the context of the transportation problem for handicapped and elderly persons.

Key words : scheduling, transit, paratransit, time-tabling

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RÉSUMÉ

Dans ce document, nous faisons une brève revue des méthodes utilisées pour la planification des horaires de transport collectif et paracollectif. Nous présentons de façon générale les méthodes utilisées en pratique et nous renvoyons le lecteur aux références fournies pour de plus amples détails. Nous considérons particulièrement les problèmes d'élaboration des horaires de service, des horaires et affectations de véhicules, et des horaires de chauffeurs. L'élaboration d'horaires par ordinateur pour le transport paracollectif est décrite brièvement dans le contexte du transport des personnes handicapées.

Mots clés : horaire, transport collectif, transport para-collectif

INTRODUCTION

The purpose of this paper is to briefly review and comment on the state of the art of schedule planning for transit and paratransit operations. The methods used in practice will be broadly reviewed and references for detailed descriptions will be given. In particular, the reader interested in all types of vehicle scheduling and crew scheduling problems should consult Bodin et al. (1983). The crew scheduling problem in transportation is also reviewed by Rousseau (1984) and the proceedings of the last two workshops on transit vehicle and crew scheduling (Wren, 1981; Rousseau, 1985) give an excellent set of references on current practice in transit scheduling.

In the next sections we examine in turn the problems of time-tabling, vehicle scheduling and crew scheduling for transit operation. Computer scheduling for paratransit will be described in the context of the transportation problem for handicapped and elderly persons.

TIME-TABLING

For the time-tabling problem we assume that a frequency plan is available by time of day. This frequency plan has either been calculated from observed data, assignment of passenger flows or more simply by marginal modification of the previous year's or period's plan. It also takes into account the company policy about the minimum level of services to be provided. An example of such a frequency plan is given in Table 1. Reading the first line of the table we are requesting that starting from 7:30, there should be a bus leaving terminus (term) plc1 (place 1) every 15 minutes (frequency) for terminus plc5 (place5). The direction of the service is indicated by the arrow. If no other indication is given, buses depart for the service in the other direction as soon as available. The running time from plc1 to plc5 at this time of the day is 29 minutes while

Table 1 : A frequency plan as found in Hastus[1] software.

HASTUS-micro

Service demand service s1

Start	Term.	Runtime	Frequency	Runtime	Term.	Start
7:30	plc1	29	15:00	30	plc5	-----→
9:00	plc1	25	20:00	27	plc5	-----→
15:30	plc1	32	15:00	30	plc5	-----→
←-----	plc1	25	30:00	28	plc5	-----→

*** end of service plc5 toward plc1 between 23:00 and 23:15

it is 30 minutes for the return trip from plc5 to plc1. The objectives of time-tabling activity are to generate a trip time-table (see Table 2) that respects the frequency plan and to generate a set of vehicle tasks, called blocks of trips, on the same route (see Table 3). Reading the first line of Table 3, we find out that the first vehicle on this route will be sign on by the driver at 7:15, will leave the garage at 7:20, will arrive at plc1 at 7:30 to start service on the route S1. It will leave the route at 9:26 from plc1 to arrive at the garage at 9:36, and it will be sign off at 9:46 for a total duration of 2h31. This information will serve as the basis for the vehicle scheduling to be discussed in the next section. For public information and drivers worksheet, the schedule could also be presented as in Table 4.

In fact, when the frequency is low on a given route, and the deadheading between route terminals is small (for example, when a set of radial routes converges toward the town center), keeping the vehicles on the same route may result in large layover (waiting) time at terminals. In this case the output of time-tabling activity may be only the set of trips.

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[1] Hastus is a product originally developed at the Centre de recherche sur les transports of the Université de Montréal distributed by GIRO Inc.

Table 2 : Partial trip time-table.

HASTUS-micro

TIME POINTS	plc1	plc2	plc3	plc4	plc5
s1	0730	0742	0748	0752	0759
s1	0745	0757	0803	0807	0814
s1	0800	0812	0818	0822	0829
s1	0815	0827	0833	0837	0844
s1	0830	0842	0848	0852	0859
s1	0845	0857	0902	0905	0911
s1	0900	0911	0916	0919	0925
s1	0920	0931	0936	0939	0945
s1	0940	0951	0956	0959	1005
s1	1000	1011	1016	1019	1025
s1	1020	1031	1036	1039	1045
⋮	⋮	⋮	⋮	⋮	⋮

Note : a similar table is produced for the service in the other direction.

Table 3 : Description of blocks of trips for the time-tables.

HASTUS-micro

Blk	Run no	Garage		Place	Start	End	Place	Garage		length
1	1	7:15	garl 7:20	plc1	7:30	9:26	plc1	9:36	garl 9:46	2h31
2	2	7:30	garl 7:35	plc1	7:45	23:19	plc5	23:25	garl 23:40	16h10
3	3	7:45	garl 7:50	plc1	8:00	23:49	plc5	23:55	garl 24:10	16h25
4	4	8:00	garl 8:05	plc1	8:15	18:51	plc1	19:01	garl 19:16	11h16
5	5	15:30	garl 15:40	plc1	15:50	20:45	plc5	20:51	garl 21:06	5h36
6	6	16:00	garl 16:10	plc1	16:20	20:15	plc5	20:21	garl 20:36	4h36

On the other hand when the frequency is relatively high, buses can be maintained on the same route as long as desired without generating large layover times at terminals. This facilitates operations and prevents the repercussion effects of large unexpected delays on a given route on the other routes of the system. In this case, the output of time-tabling activity should be blocks of vehicle trips that can be combined further by the vehicle scheduling activity into vehicle duties.

Table 4 : Trip time-table for public information.

HASTUS-micro

Route	s1	s1	s1	s1	s1	s1	s1	s1	s1	s1	s1
Brady Square.....	0730	0745	0800	0815	0830	0845	0900	0920	0940	1000	1020
Washington Village.....	0742	0757	0812	0827	0842	0857	0911	0931	0951	1011	1031
Washington Bus Station.....	0748	0803	0818	0833	0848	0902	0916	0936	0956	1016	1036
Concord.....	0752	0807	0822	0837	0852	0905	0919	0939	0959	1019	1039
Coach Road Estate.....	0759	0814	0829	0844	0859	0911	0925	0945	1005	1025	1045

Combining trips into blocks is generally fairly trivial. After ensuring that a minimum layover is allowed to maintain the regularity of services (Wilson, 1986), a first in first out (FIFO) strategy is optimum on a route by route basis both for the number of vehicles necessary and for the minimization of total duty times (Saha, 1970). A vehicle is freed as soon as it is not required any more or would have to wait an excessive amount of time for the next trip assignment. A vehicle is dispatched from the depot when needed for the trip schedule.

Occasionally, the scheduler may want to lengthen or shorten a block either because it is too long for operations without refuellings or because it is shorter than the minimum duration of a driver's piece of work[1]. In this latter case it could generate a penalty cost in the driver schedule. A simple exchange of times at the terminal between two adjacent departing vehicles can be performed easily to achieve this aim. All adjacent departure trips in the block to be modified must be examined to determine the exchange that achieves this aim while increasing waiting time the least.

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[1] A piece of work refers to the period of time during which a driver works continuously with the same vehicle without a break. Generally the number of pieces of work in a duty is limited (2-3 pieces).

Table 5 describes an example where exchanging trips between vehicles reduces the penalty for short blocks by one hour, while creating only an additional 5 minutes of waiting time.

Table 5 : Exchange of trips to reduce penalty cost for driver assignment.

Solution 1

	Block starting time	arriving term A	departing term A	Block ending time	Block duration	Penalty
Vehicle 1	7:55	8:55	not required anymore	8:55	1h00	1h
Vehicle 2	6:00	9:00	9:05	18:20	12h20	—
				Total	13h20	

Solution 2 (after exchange)

	Block starting time	arriving term A	departing term A	Block ending time	Block duration	Penalty
Vehicle 1	7:55	8:55	9:05	18:20	10h25	—
Vehicle 2	6:00	9:00	not required anymore	9:00	3h00	—
				Total	13h25	

There are very few mathematical considerations in time-table preparation, but there is a need for a sophisticated management information system. Several bus scheduling software packages are available, most of them integrated into a crew scheduling software (see Rousseau, 1985). For example, Hastus facilitates the input or modification of the frequency plan and of the data related to travel time and minimum layover time on the route according to the time of day, day of week and season of year. In addition the schedule generated respects the first departure time specified, allow the definition of an interval for the last departure time

and is able to handle several termini on the same route. It produces time-tables at various reference points on the route for the preparation of public information. It also automatically smooths the variations in trip times and in headways. The specified frequency plan may specify headways between two trips that may not divide exactly into the time period to which the plan applies. For example :

From 9:30 headway 12 minutes
From 10:00 headway 30 minutes

By smoothing the differences in headways HASTUS generates departures at 9:30, 9:42, 9:54, 10:15, 10:45, etc. for intervals of 12, 12, 21, 30 minutes, etc.

Finally, it is often necessary to look at two or more routes that include the same segment of service in order to better synchronize their activity on that segment. Hastus let the scheduler look at the time-table of several routes at the same time at given reference points. Graphical representations of route schedule are used in some European cities (Nantes, for example) to better synchronize services on the same route segment.

We believe that in the future there will be more development in the area of graphical representation of schedules by computer to facilitate the synchronization of bus route schedules and reduce transfer time. For example, very good graphical systems have been developed for rail operations (Rivier and Tzieropoulos, 1985). We expect also that in the future, the update of the necessary data for time-tabling and frequence plan development will be obtained directly from automated vehicle monitoring system including passenger counting devices.

VEHICLE SCHEDULING PROBLEMS

The simple case

By vehicle scheduling, we refer to the class of problems in which vehicles must be assigned to time-tabled trips, or tasks in such a way that

each task is carried out by one vehicle, a given set of constraints is satisfied and a cost is minimized. For simplicity here, the term "trip" will be used to denote all vehicle tasks defined in the time-tabling activity (whether trips or blocks of trips). Very often the cost structure reflects the fact that the number of vehicles must be minimized. We assume that for each trip, the departure time, trip duration, origin and destination terminal are known. In addition to regular trips, we assume that deadheading trips are allowed between terminals.

The problem where no deadhead trips are allowed was addressed in the previous section when we discussed the formation of blocks. In this section we will discuss the problem as stated above with one depot. We will then consider in turn some complicating constraints. The reader who requires more details on this problem could consult Scott (1986), Carraresi and Gallo (1984), Bodin et al. (1983) and Ceder and Stern (1981).

In a recent paper Scott (1986) presents an excellent survey of the development of approaches for this problem. We follow his description.

The earliest attempts to solve the vehicle scheduling problem focussed on the minimum fleet size necessary to serve a fixed set of trips. Dantzig and Fulkerson (1954) were the first to pose this problem as a linear program. Later, Dantzig and Hoffman (1956) recognized a parallel structure between this scheduling problem and that of finding a minimal chain decomposition of a partially ordered set (Dilworth, 1950). Ford and Fulkerson (1962) drew a further parallel with the König-Egervary theorem in graph theory and proposed a simple solution algorithm. Raghavachari and Mote (1970) extended this theory to the problem of minimal chain decomposition of a directed acyclic graph. Such a graph can be constructed without initially assuming a partially ordered set. These authors give public transport vehicle scheduling as an example of such a chain decomposition on an acyclic graph.

A somewhat different line of inquiry into the fleet size problem has examined the use of "deficit functions" (Gertsbach and Gurevich, 1977;

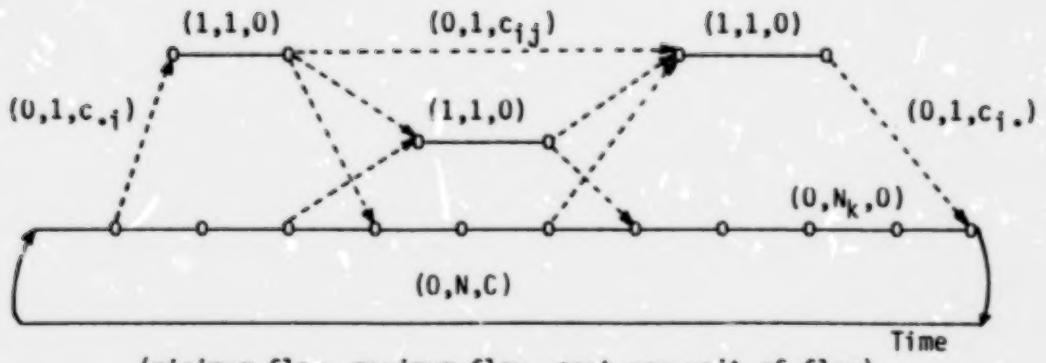
Ceder and Stern, 1981; Stern and Ceder, 1983). A deficit function is a step function defined at a terminal (a place where trips start or end). Each time a trip leaves a terminal, the deficit function increases by one at that terminal, and each time a trip arrives at a terminal the deficit function decreases by one. Following Bartlett (1957) and Salzborn (1972), Ceder and Stern (1981) use deficit functions to obtain upper and lower bounds on the total fleet size required to serve a schedule and propose a heuristic for assigning vehicles to trips in such a way that the fleet size is minimized. Ceder and Stern (1981) note that under general conditions these heuristics are equivalent to Ford and Fulkerson's (1962) algorithm. The deficit-function approach has the advantage, however, of being oriented toward graphical representation, which provides the basis for straightforward manual modifications of a vehicle solution. Such modifications are necessary when an automatic solution violates additional constraints such as those on manpower usage or the total length of a vehicle path, as polynomial-time exact solution techniques do not yet exist for these problems (see, e.g., Bodin et al., 1983).

Following the tradition of linear-programming approaches to the vehicle scheduling problem, several authors (Bennington and Rebibo, 1975; Orloff, 1976; Gavish et al., 1978) have presented models which minimize objectives more complex than just fleet size. These models are motivated by the fact that in addition to fleet size, the time that vehicles spend away from the depot and distance travelled during the course of a schedule also contribute to the cost of serving a set of trips. The linear programs presented by these authors each represent a problem of minimum-cost flow on an acyclic graph and allow nonnegative costs on all arcs in the graph. Like the chain decompositions described above, a solution to this problem of miminum-cost flow is a decomposition of the set of trips into chains of trips, each of which is to be served by a single vehicle. In order to accomodate fleet size into the model, however, a large cost is associated

with each complete chain. This "fleet size penalty" is generally set several orders of magnitude larger than other arc costs, to reflect the preponderance of vehicle capital costs in the vehicle scheduling problem. The minimum cost flow problem set up in this manner will ensure that fleet size is minimized, as a large cost accompanies the entry of each vehicle into service. The secondary objective of minimum operation cost is, however, met through the use of realistic arc costs.

Such linear-programming approaches to the scheduling problem have had a certain appeal because of their conceptual simplicity and ease of implementation. In practical scheduling problems, however, a model which minimizes the number of vehicle chains may not serve the real goals of optimization. This is because a single vehicle could conceivably cover two or more chains of trips. In the case of time-tables in which the frequency of service varies with time of day, a vehicle solution in which vehicles are removed from service during periods of low trip frequency will normally reduce manpower costs and corresponds in general to the kind of solution favored by transport operators. Scott (1984) has pointed out that a model which applies a fleet size penalty to each chain can induce a solution with artificially inflated vehicle time and distance, because the model includes a built-in bias against the creation of independent chains. Such a result is something of a distortion of the original scheduling objectives.

Desrosiers et al. (1982) have presented a scheduling model which takes into account the use of a single vehicle to serve many chains of trips. Figure 1 describes the approach proposed. In this approach, each trip in the schedule is represented by an arc with a minimal and maximal flow of one thus requiring a unit flow to traverse it. There is no specific cost associated with the trips themselves because they all have to be used. All feasible deadhead trips are also represented by arcs with a minimum flow of zero and maximum flow of one and an associated cost c_{ij} corresponding to the cost of deadhead time and distance. Equivalently, a



(minimum flow, maximum flow, cost per unit of flow)

o——o : correspond to trips
- - - - : deadhead trip to or from the depot or between regular trips

Figure 1 : A time space network for the transit bus scheduling problem.

cost is associated with a deadhead trip to and from the garage ($c_{i.}$,$c_{.i}$). The originality of this formulation lies in the fact that the garage is not represented by a single node but by a sequence of time nodes. All buses in the garage can flow freely from a time node to the next time node without cost. A bound N_k on the maximum number of buses in the garage at time period k can be imposed easily on the solution. Finally a return arc from the end of the day to the beginning of the day ensures the conservation of flow and counts the number of buses required to operate the schedule. The fixed cost C of a vehicle and an upper bound N on the number of vehicles can be imposed on this arc.

A standard minimum cost flow algorithm can be used to solve this problem. This formulation accurately takes into account the number of buses required without generating too many arcs.

However Scott (1986) has demonstrated recently that if deadhead cost is proportional with travel time and waiting time, which are characteristic of a broad class of vehicle scheduling problems, there need not be any conflict between the two objectives of minimizing fleet size and minimizing

the cost of operating a schedule. In other words, there is never, in general, any need for the application of a fleet size penalty to the creation of independent chains when one intends to find a minimum-cost scheduling solution which uses a minimal fleet.

Complicating constraints. We will consider here three types of complicating constraints that can destroy the structure of the basic vehicle schedule observed earlier. First we may want to impose a minimum length or a maximum length on the duration of a vehicle duty (set of trips assigned to a vehicle between its visits to the garage). The minimum length constraint arises in the context where there is a minimum duration for each piece of work assigned to a driver, for example 2 hours. In this context, a vehicle duty of duration less than two hours will cost a minimum of two hours of driving time. It is thus advisable to try to reduce the number of such vehicle duty as far as possible. The maximum duration constraint is in general related to the necessity of refuelling vehicles.

Both of these problems have to be dealt with heuristically. In practice the following heuristics have been found useful :

Step 0. Solve the problem without these maximum or minimum length constraints.

Step 1. Split the day into a certain number of time periods.

Step 2. For each time period split the vehicle duty in two partial duties : set of trips finishing before or during the time period, and the set of trips finishing after the time period. Solve an assignment problem between the two sets of partial duty taking into account the cost of having vehicle duty durations inferior to the lower limit, and prohibiting duties from being longer than the upper limit.

Step 3. Cycle through the time period or various sets of time periods until no improvement is possible.

The second set of complicating constraints arises because one may want to take advantage of a certain level of flexibility in the time-table concerning the departure time of the trip (for example, trip departure time may vary within ±5 minutes of a desired time). Desrosiers et al. (1984) have described a combinatorial approach to this type of problem in which they solve a set covering problem with column generation over all possible feasible vehicle duties respecting the time windows. These vehicle duties are not pregenerated but are generated as the algorithm progresses using shortest path or dynamic programming type algorithms.

A third set of constraints occurs for companies that operate their fleet out of several garages each with a preset capacity. In this case, they require that a vehicle belonging to one garage must return to the same garage. Additional constraints may also be imposed. The company may want to use each garage at a minimum capacity even if this is not cost effective as is very often the case with new garages that have to be located in industrial rather than residential areas. The company may also require that a certain number of vehicles be always out of the garage to facilitate maintenance activities. Finally for control reasons, the company may not accept that the vehicles assigned to a route may be split between several garages.

Various approaches have been proposed for this problem (Bodin et al., 1983; Mandle, 1980; Carraresi and Gallo, 1984; Désilet and Lessard, 1984). Essentially a choice must be made between clustering the trips and assigning them to the garage first, and then scheduling vehicles in each garage separately, or first scheduling the whole fleet as if there were only one garage and then assigning the resulting trips to each garage. If there are no constraints on separating each route among several garages, Carraresi and Gallo (1984) have proposed an approach based on a Lagrangean relaxation of the problem coupled with a subgradient optimization technique to find an optimal solution. On the other hand, Désilet and Lessard (1984)

have proposed a simple clustering heuristic that can take into account all kinds of operational constraints. They sequentially assign all the trips of each route to the best garage and then search for a feasible solution by neighbourhood exchange heuristics, moving routes from one garage to another in order to improve the cost or feasibility in relation to the various constraints. They then optimize the schedule for each garage. These methods can also be used to study the location of new garages and evaluate their impact on operational costs.

Finally, even if crew costs represent often more than 60% of operating costs, it has not been found very useful in general to take into account the crew scheduling constraints in preparing the vehicle schedule. The minimum length of a piece of work seems to be the only important constraint to be taken into account at this step. We believe that only in very specific contexts such as small problems with particular constraints would it be profitable to take these constraints into account. This problem has been studied by Scott (1984).

CREW SCHEDULING

Crew schedule planning in a transit company involves two different operations : the generation of the drivers daily duties and the assignment of a set of daily duties (called a roster) to each driver. We will consider each of these problems in turn. The proceedings of the last two workshops on computer scheduling of public transport constitute an excellent set of references on the subject (Wren, 1981; Rousseau, 1985).

Duties Generation

The problem can be expressed simply as the creation of a set of duties of minimum cost ensuring that each bus trip is covered by a driver (or a crew), and that all union contract rules are respected. Several authors favour an approach based on the set covering formulation of the problem. Theoretically, in this approach all feasible duties have to be

generated according to union contract rules and the least cost subset of duties that covers all bus trips is mathematically selected.

If the operation or union rules restrict a driver to working on only one bus route, and if there are not too many feasible duties due to strict working rules, this approach may be feasible. Otherwise, if for example full interlining[1] is permitted, the number of feasible duties is extremely large and just cannot be generated. Advocates of the set covering approach then have to either split the problem into several subproblems (evening problem, morning problem, etc., see Ward et al.,1981) or heavily restrict the generation of duties according to heuristic rules. This is the case of Busman, Ramcutter, Crewsched, etc. (see Rousseau, 1985, for several papers based on this approach).

The difficulty with this approach is mainly that there does not exist a good set of rules to preselect the good feasible duties. A typical duty may be very efficient under certain operation and union rules while being very inefficient under another set of rules. A very interesting study has been completed recently by Desrochers (1986) where, while using the set covering approach for the problem, he sequentially generates only the duties useful to improve the solution at each step of the algorithm. Thus only a small subset of the duties have to be generated. It seems however that the technique will apply to only small to medium size problems but could be quite useful in the context where the drivers have to work on only one bus route.

The most popular approach for transit crew scheduling proceeds as follows.

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[1] Interlining for drivers means that they might work on several bus routes during a single day operation. Interlining for buses means that buses may change routes withing the same vehicle duty.

First, the vehicle duties, called blocks[1] in the literature, are split into pieces of work[2]. Secondly, the pieces of work are matched together to form driver duties and thirdly the solution is improved heuristically and manually. A matching or an assignment problem formulation for the construction of duties is generally used. Various strategies are possible for splitting the blocks into pieces of work. Most approaches use heuristics adapted to the type of problem considered. In RUCUS (Wilhelm, 1975), the first general package developed for transit crew scheduling, a set of parameters enables the user to partially determine the method by which the pieces will be defined. In Sage runcutting and RUCUS II (Luedtke, 1985), an interactive system for transit crew scheduling, the user has direct control on the ways the pieces are identified. Hoffstadt (1981) and Parker and Smith (1981) include heuristic approaches adapted to their respective problems related to German and British working conditions. The HASTUS system uses a relaxation of the problem to determine how to define the pieces. Bail et al. (1981) form the pieces and blocks by sequentially matching the trips.

While using this general decomposition approach, HASTUS uses a mathematical model to split the blocks into pieces of work. A full description of the algorithm can be found in Rousseau et al. (1985) The authors chose to solve a relaxation of the crew scheduling problem first, trying to keep the main features of the problem while relaxing all the details.

A relaxation of the drivers' scheduling problem is solved on a modified bus schedule. The idea is to retain the essentials of the bus

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1 In the crew scheduling problem, the block is generally defined as the itinerary of a vehicle between its departure from and its return to the garage. It includes all deadhead times to take the bus out of and into the garage to and from the route(s) served.

2 Relief times are the times corresponding to places on a route where a change of drivers is possible. In general the number of such places on each route is small.

schedule structure and its impact on the contract rules and costs without retaining the precise bus schedule.

Firstly, periods defined by the user are introduced. These periods are normally 30 minutes long; they can vary, depending on the problem, between 15 and 60 minutes. The schedules of all buses are approximated in order that all buses start their block at the beginning of a period and complete it at the end of a period. The authors further assume that relief times[1] can only occur at the beginning of a period. Moreover, the problem is further relaxed by requiring that the duties selected be sufficient to cover the total requirement of drivers per period instead of requiring that they exactly cover all the blocks individually.

All feasible duties satisfying the union contract in use, two or three pieces of work covering an integer number of periods are generated. The generating rules eliminate, under instructions from the user, duties which are of no interest or which do not correspond to unwritten rules or common practice (for example, a duty with a long break during a peak hour). All the duties are costed according to the union contract in use. A linear program then searches for the set of duties that will provide the number of drivers required during each period and satisfy other additional constraints at minimum cost.

One type of additional constraint is related to the union contract. All the constraints related to the total number of duties of a certain type are considered. These include constraints on the minimum or maximum number of duties without break and constraints on the maximum number of duties with certain undesirable characteristics like long spread[2], small breaks, etc.

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[1] Relief times are the times corresponding to places on a route where a change of drivers is possible. In general the number of such places on each route is small.

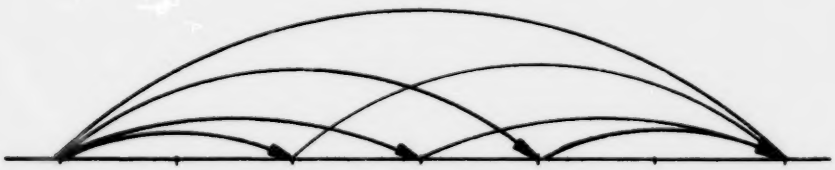
[2] The spread is the total length of the duty from the sign on to the sign off. It includes all unpaid breaks.
Example : duty - 1rst piece of work - 6:00 to 10:00
- 2nd piece of work - 14:00 to 18:00
The spread is 12 hours.

The relaxation is called HASTUS-macro and can also be used as a planning tool for the evaluation of the economic impact of changes in the union contract or in the service level. By changing the cost parameters or the service level a new solution is obtained and can be compared with the current solution. The paper by Blais and Rousseau (1982) describes in detail the HASTUS-macro system and its use in that context. In practice, this relaxation has been found to be very efficient, producing cost estimates within 1% of the real operating costs for drivers.

The solution of HASTUS-macro gives an indication of the types of duties that should be constructed in a final solution. The next step uses this information to split the blocks into pieces of work that correspond as closely as possible to the ones generated in the relaxation HASTUS-macro.

Each block is sequentially split into pieces of work by solving a shortest path problem. It is interesting to note that the problem of splitting a block into pieces can be expressed as the problem of finding a path from the starting time of the block to its ending time using arcs that correspond to pieces of work. Figure 2 illustrates this concept.

The feasible pieces of work (we are assuming here a minimum length of two hours) are represented by arcs and listed in the figure. We have assumed for simplicity that a relief time exists at every hour in this example.



The feasible pieces are

7 - 9	7 - 11	9 - 11	10 - 13
7 - 10	7 - 13	9 - 13	11 - 13

Figure 2 : The flow formulation for the partition of a block.

The cost associated with each piece of work in the flow formulation corresponds to a penalty indicating whether this given piece will increase or decrease the difference between the number of pieces of work of that type desired, and the number of pieces of work of that type already present in the current solution. We cycle through all blocks until no improvement can be achieved towards coming closer to the number of desired pieces of each type indicated by HASTUS-macro.

Once the blocks have been splitted into pieces of work a matching algorithm is used to generate duties (Rousseau et al., 1985) and several heuristics are used to improve on the solution. The whole system is embedded in a user friendly software (Rousseau and Blais, 1985) and is installed in several cities around the world (Montreal, Quebec, Calgary, Seattle, New York, Boston, Stockholm, Singapore, Newcastle, ...). The typical savings of this system over a manual solution is of the order of 15% to 20% of unproductive costs (unproductive time, overtime, special payments, etc.), which could represent 2-3% of total operating costs (for example, approximately 3 million dollars of savings per year in Montreal).

The rostering problem. A roster can be defined as the set of duties assigned to a particular driver. Making up rosters may be easy or difficult depending on the particular union contract. In most North American companies the assignment of duties is left to the drivers themselves and the decision is made on a seniority basis : this leaves little or no room at all for mathematical modelling! On the other hand, with many European companies, work load must be distributed evenly among the drivers. In fact in these problems, few penalties are associated with unpleasant duty conditions. Rather, these duties are evenly distributed among all drivers, who all receive approximately the same monthly pay. The rules for making monthly, weekly or in some cases several month rosters vary widely from one company to another. While there seems to be no general package available

for rostering problems in transit, there is a consensus on the most efficient methods to be used to build such rosters. A more detailed description of the approach used can be found in Belleti et al. (1985), Hagberg (1985) and Rousseau (1984).

The basic method is a heuristic based on a construction phase and on a marginal improvement phase. The construction phase can be described as follows : given a set of partial rosters P_i constructed from day 1 to day i (initially the set of partial rosters is composed of all duties for day 1), construct P_{i+1} by defining an assignment problem between the partial rosters in P_i and the duties that have to be performed on day i+1.

The assignment problem can be solved with standard well known algorithms. The objective function is a penalty function measuring how the resulting partial rosters will differ on various criteria from the ideal partial rosters. For example, if the main criterion is to equalize as much as possible the total number of driving hours among the rosters, the penalty incurred by assigning duty k to partial roster j might be the square of the difference between the number of driving hours in the resulting roster and the desired level for such a number after i+1 days. A general penalty function is described in Nicolletti (1973 and 1975) for the airline rostering problem.

As for the marginal improvement, all kinds of exchange algorithms well-known in routing and scheduling can be used in this context. An interesting approach however has been applied by Carraresi and Gallo (1984) for the transit crew rostering problem in which the objective is to minimize the maximum total driving time in the period considered. This problem is described as the multi-level bottleneck problem and it is shown that the problem is NP-complete. An asymptotically optimal algorithm is presented for this case. In general the objective function is different but their type of algorithm could easily be adapted to the general situation when starting from a feasible solution.

Sequentially consider each day ℓ in the rostering period.

Option 1 : Remove from each roster the duties of day ℓ. Define and solve an assignment problem between the duties just removed and the partial rosters with the penalty function in use, allowing only feasible assignments.

Option 2 : Define and solve an assignment problem with on one side the partial rosters composed of all the duties starting before day ℓ and on the other side the partial rosters composed of all the duties starting on or after day ℓ. Of course only feasible assignments are allowed and the same cost function as in the construction heuristic is used.

If no improvement in the objective function is obtained after all days in the rostering period have been considered the process stops. If an improvement has been achieved, another iteration is performed on the period. From the results of Carraresi and Gallo on their specific problem we can expect this algorithm to perform very well on all rostering types of problems even if the theoretical properties demonstrated by the authors may not hold in all situations. These methods obviously need to be adapted to particular situations, but they have been used successfully in several contexts.

COMPUTER SCHEDULING FOR PARATRANSIT SERVICES FOR THE ELDERLY AND THE HANDICAPPED

The operation of specialized transit services for the elderly and the handicapped involves complex registration, reservation, scheduling and dispatching tasks and the production of numerous management reports for internal and external use. This context, combined with the need to provide more service per dollar, has led many companies to consider computerized aids. We present here some general conclusions (see Hamer, 1986) on the design, installation and use of computer-assisted paratransit systems drawn

from practical experiences in Toronto, Montréal and Sherbrooke rather than present specific scheduling algorithms.

The Montreal system, installed on a large IBM mainframe, permits advance and demand-responsive[1] scheduling while the client is waiting on the phone. The Toronto system, installed on a dedicated minicomputer, is geared to advance scheduling followed by a call- back process to confirm the time of pick-up. Both systems use scheduling algorithms based on recent work by the Centre de recherche sur les transports of the Université de Montréal funded by Transport Canada (Roy et al., 1985). These are large-scale operations, handling between 1200 and 2000 trips per day and serving large metropolitan areas. The Sherbrooke system is typical of requirements for smaller companies (200 trips per day). It is being implemented on a micro-computer with a hard disk. These systems were developed by GIRO Inc. in cooperation with the respective transit companies. Major funding has been provided by the Quebec and Ontario governments.

The functions of a computer system for specialized transit can be categorized as follows : client registration, trip reservation and confirmation, trip scheduling, dispatching, operations reporting, management reporting, and financial functions. The various systems in operation in Canada (Geehan, 1985) permit partial or full automation of some or all of these functions. However, the relative emphasis placed on the various functions varies greatly. For example, in cities where several vehicle operators or taxi companies provide the actual service, operations reporting and financial functions tend to be emphasized. For smaller operations, the scheduling task is often less automated than in larger

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[1] Subscription trips refer to regular trips registered for an unlimited period of time mostly for work or study reasons. Advanced booking refers to a trip requested several days in advance while demand-responsive trips are requested for the current day.

cities : in the latter case, the size of the demand necessitates a coordinated effort between several schedulers. The mix between subscription trips, advance bookings and demand-response trips, as well as the presence or absence of a confirmation process, greatly affect the type of system required. Finally, a series of details have important consequences on scheduling software : do schedulers and dispatchers provide both pickup and drop times to drivers, or do they only provide pickup times, leaving the detailed routing to the driver?; is the service forced to refuse a significant number of requests because of insufficient vehicle availability or are almost all requests honored?; what kind of service flexibility is allowed between requested trip times, scheduled times, and effective on-the-road times?

Each locality therefore has a unique mode of operation based on client service policies, the size of the demand, funding levels and policies, and the division of responsibilities between public transit authorities, private bus operators, taxi companies and government agencies. The bottom line is that, except for the simpler functions, it is impossible to define a generic software system that will suit everyone : at the least, a significant amount of customization is necessary for each installation of an established package.

We believe that a computerized system should permit some development in the level of automation provided, by the addition of optional features and functions in the following order :

- computerization of the client database;
- computerized reservation with manual scheduling using computer printed trip request forms ("trip tickets");
- computer-aided scheduling using a route suggestion algorithm; the computer suggests several routes into which the new trip request may be inserted but the scheduler takes the final decision;

- automated scheduling on a trip-by-trip basis using a detailed zone-to-zone travel matrix; the computer automatically inserts the new trip requests one at a time in a route;
- global route optimization using advanced algorithms where the computer organizes or reorganizes the routes given all requests (Desrosiers et al., 1984b).

This approach has the advantage of being open-ended and flexible and we feel that it offers the best guarantees for evolutionary growth as paratransit services expand and as more experience with computer-assisted scheduling is gained. Experience has also shown that the introduction of complex and costly scheduling systems must be preceded by the installation of a sound data processing system, including the generation and validation of client, trip and travel time data. Computer response time, for example, is critical : the most advanced algotihms are of little use if execution time is excessive.

In addition, the introduction of automatic computer scheduling is always expensive not because of the algorithms involved but because of the data necessary for this task. The computer must be able to estimate with precision the travel time between two pick-up or drop off points. These points must be localized automatically with some precision and a good travel time function must be calibrated. For example, Toronto has been split in approximately 1000 zones. Using a street address data bank, a program automatically locate a given address into a zone helping the user to input sufficient informations for a unique match. A distance matrix on the network was calculated from each zone to each zone using the actual geocoded street network (Statistics Canada data was used) and a travel time matrix was obtained from a speed fonction varying with the distance.

It is common adage of the computer business that it is essential to clearly define one's needs and goals before proceeding with system design or procurement. What are these goals for specialized transit and how can we measure, quantitatively, our success in achieving them? Globally, the

thrust is to provide the best quality service at a reasonable cost, but this statement of goals is both too general and too subjective. More specific measures include the following :

a) vehicle productivity in trips/hour, customers/hour, or customers/kilometer;

b) cost per customer trip in dollars, including on-the-road costs, reservation and scheduling costs, and administrative overhead;

c) average telephone dwell time for the trip reservation process;

d) scheduling lead time (how long does it take to schedule and confirm a reservation or a batch of reservations?);

e) service quality measured firstly, by the difference between times confirmed to customers and effective pickup and drop off times, and secondly, by the number of scheduling errors or omissions;

f) percentage of rejected trip requests;

g) lead time and person-hours required to produce daily operating statistics.

Some of these measures are controversial conflictual and somewhat ill-defined and comparisons from one company to another are often difficult. Relative measures, however, taken before, during and after computerization of specific functions can and should be used to judge effectiveness. Other more subjective criteria, such as system acceptance by drivers, office personnel and management, should also be evaluated.

The computer has earned its place in most large handicapped and elderly transportation services and in several smaller ones, and while the field is far from mature, a few systems with proven effectiveness and varying levels of automation are now available on the Canadian market. In particular Montreal and Toronto systems have demonstrated the usefulness of the computer in the scheduling function by helping to produce better schedules in less time.

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